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A data transmission scheme with spectral switches of a shifting double slit in the far field

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Abstract
A novel data transmission scheme that can be used for any desired transmitting direction in free space is introduced. This scheme is obtained by analyzing the analytic form of the diffracted spectrum of a coherent broadband Gaussian spectrum incident on a shifting double slit. The merits of this method are that the transmission angle can be chosen arbitrarily in advance and that it is easy to implement compared with other previous schemes which modulate some properties of the light source.

Keywords: singular optics, spectral switches, shifting double slit, Gaussian spectrum, Fresnel–Kirchhoff diffraction integral, data transmission scheme

1. Introduction

Spectral anomalies [1–13], which are induced by the diffraction of an aperture for a polychromatic light source (or broadband pulses), have lately gained more interest because of their different applications such as in lattice spectroscopy [1] and spatial coherence spectroscopy [4]. Another important application is to transmit the digital information in free space with the so called spectral switches phenomenon [2, 3, 5–7] which has been verified experimentally [11–13]. In the past, the spectral switch phenomenon was attributed to the singular optics effect in which drastic spectral changes take place near some singular points with zero amplitude [5–7]. However, we have shown that spectral switches can exist without phase singular points and the correct relationship between them was clarified [2]. It is found that the singularities are only a sufficient condition for the spectral switches and that the necessary condition is the oscillatory behavior of the modifier function. However, most of the previous methods [5–7, 9, 10] provided only numerical results and the transmitting directions were unpredictable; thus their practical uses are seriously limited. For example, in [7], the Fresnel diffraction integral was integrated directly to investigate the spectral behavior in both near field and far field, and the transmitting angles can only be found numerically. In this work, we present a novel scheme such that the needed transmitting direction can be specified in advance. Also it is easy to perform the control mechanism by just shifting one of the slits, instead of by changing the properties of the light source (e.g. spatial coherence or spectral bandwidth).

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2. Theory

Consider a spatially completely coherent light, with a spectral scalar field $U'(p', \omega)$, incident from the left upon a double slit where one of the slits is movable, as indicated in figure 1(a) where the right slit is assumed movable. As shown in figure 1(b), each slit has width b and their centers are positioned at $-a$ and d for the left and right slit respectively. Consequently the light wave will be diffracted and arrive at the observation (or detection) plane at (x, y, z) in the far field. The diffraction field $U(p, \omega)$ on that plane can be obtained from the

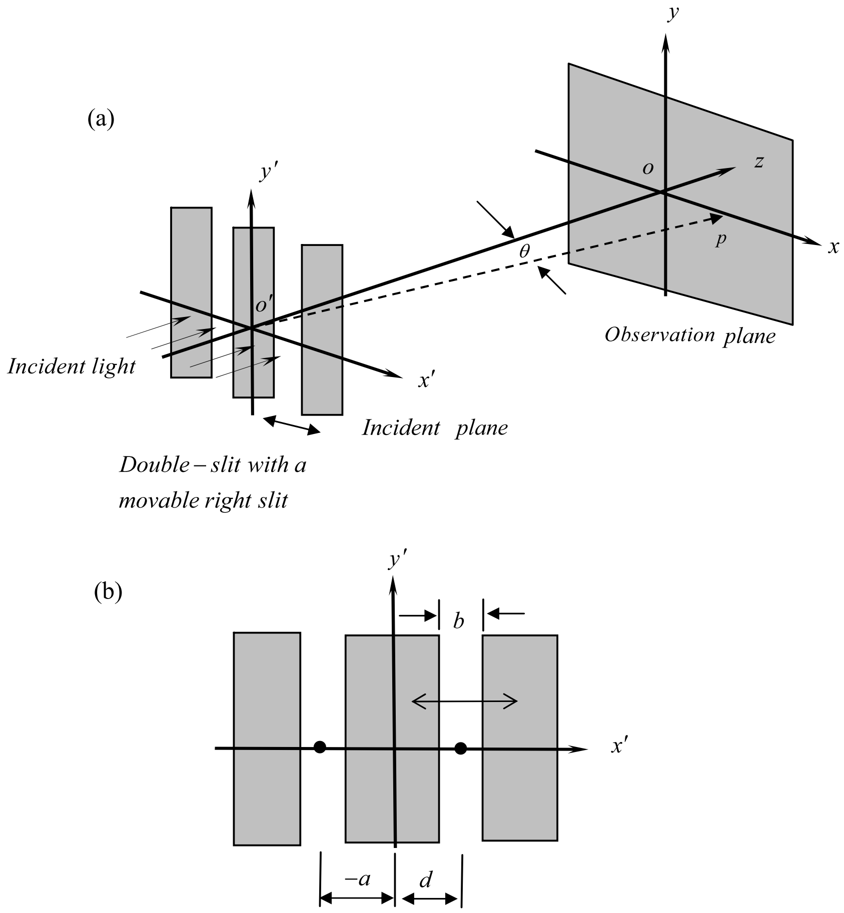

Figure 1. (a) Basic geometry. An incoming light wave, from the left, is incident on a double slit with a movable right slit. (b) Dimensions and structures of the double slit.

Fresnel–Kirchhoff diffraction integral [14] as

$$U(p,\omega) = \frac{1}{\mathrm{j}\lambda}\int\int_{\Sigma'} U'(p',\omega)\frac{\exp(\mathrm{j}\omega r/c)}{r}\chi(\theta)\,\mathrm{d}\sigma', \quad (1)$$

where $\chi(\theta)$ is the obliquity factor, λ is the wavelength, ω is the angular frequency, c is the velocity of the light wave, and r is the distance from point $p'(x', y', 0)$ on the aperture plane to point $p(x, y, z)$ on the observation plane. As plotted in figure 1(a), the coordinate systems $x'o'y'$ and xoy are used for the incident (aperture) plane and the observation (detection) plane, respectively. In the integral of equation (1), Σ' is the aperture function and $\mathrm{d}\sigma'$ is the related differential. Due to the symmetry property along the y' axis in our optical setup, we can limit our discussion to along x' and choose the observation point p along x without losing generality. In the figure, θ designates the angle between $\overline{o'p}$ and optical axis $\overline{o'o}$ as denoted in figure 1. Equation (1) is usually used for a monochromatic incident field, but it is also applicable for a broadband pulse or polychromatic field [15], which can be superposed with a monochromatic field via the Fourier integral.

The aperture function in figure 1(b) which represents the limited area of incoming light can be written as

$$g(x') = \Pi\left(\frac{x'+a}{b}\right) + \Pi\left(\frac{x'-d}{b}\right), \quad (2)$$

where $\Pi(x')$ is the rectangular function defined as $\Pi(x'/b) = 1$ for $|x'| \leqslant b/2$ and $\Pi(x'/b) = 0$ for $|x'| > b/2$. The Fourier transform of this aperture function is

$$F(g(x')) = b\,\mathrm{sinc}(\pi b f_x)[\exp(\mathrm{j}2\pi a f_x) + \exp(-\mathrm{j}2\pi d f_x)], \quad (3)$$

where the sinc function is defined as $\mathrm{sinc}(x) = \sin(x)/x$; f_x is the spatial frequency variable. We assumed that the incident spectral scalar field $U'(p',\omega)$ is spatially completely coherent light consisting of a single line of Gaussian profile, centered at angular frequency ω_0 with room mean square (rms) bandwidth Γ; that is,

$$U'(p',\omega) = \exp\{-(\omega-\omega_0)^2/2\Gamma^2\}. \quad (4)$$

Using $r \simeq [z^2 + (x - x')^2]^{1/2} \approx z + x^2/2z - xx'/z$ for the far field approximation and substituting equation (4) in (1), the diffraction field $U(p,\omega)$ can be obtained as [16]

$$U(p,\omega) = \frac{1}{\mathrm{j}\lambda z}\exp\left[\mathrm{j}k\left(z + \frac{x^2}{2z}\right)\right] \times U'(p',\omega) \times F(g(x')), \quad (5)$$

where the wavenumber is written as $k = \omega/c = 2\pi/\lambda$ and the last term $F(g(x'))$ is the Fourier transform of the aperture function $g(x')$ in equation (3) with the spatial frequency $f_x = x/\lambda z$. Substituting equations (3) and (4) into (5) with the help of equalities $1/\lambda = \omega/2\pi c$, we have

$$U(p,\omega) = \frac{a}{\mathrm{j}z}\left(\frac{\omega}{2\pi c}\right)\exp\left[\mathrm{j}k\left(z+\frac{x^2}{2z}\right)\right] \times \exp\{-(\omega-\omega_0)^2/2\Gamma^2\}\{b\,\mathrm{sinc}(\pi b f_x) \times [\exp(\mathrm{j}2\pi a f_x)+\exp(-\mathrm{j}2\pi d f_x)]\}. \quad (6)$$

With the relations $\tan\theta = x/z$ and $f_x = \omega x/2\pi c z = \omega\tan(\theta)/2\pi c$, the spectral intensity $I(\theta,\omega)$ along the x axis with angle θ can be obtained through $I(\theta,\omega) = |U(p,\omega)|^2 = U(p,\omega)U(p,\omega)^*$ as

$$I(\theta,\omega) = A\exp\{-(\omega-\omega_0)^2/\Gamma^2\}\omega^2 \times \left\{\mathrm{sinc}^2\left(\frac{b\tan(\theta)\omega}{2c}\right)\cos^2\left[\frac{(a+d)\tan(\theta)\omega}{2c}\right]\right\} \equiv AG(\omega)M(\theta,\omega), \quad (7)$$

where $A = b^2/(\pi c z)^2$, $G(\omega) = \exp\{-(\omega-\omega_0)^2/\Gamma^2\}$ is the spectrum of the incident light source, as derived in equation (4), due to $G(\omega) = |U'(p',\omega)|^2$, and $M(\theta,\omega) = \omega^2\{\mathrm{sinc}^2(b\tan(\theta)\omega/2c)\times\cos^2[(a+d)\tan(\theta)\omega/2c]\}$ is called the modifier function. As indicated in equation (7), this modifier function illustrates how the spectrum of the light is modified (or modulated) as a result of diffraction at the aperture. Equation (7) is used to give some numerical examples below that characterize spectral anomalies and the spectral switches in different situations.

3. The scheme for data transmission at any angle θ

From equation (7) and figure 1, we can discuss how the diffracted spectrum varies as the detection angle changes. Also we devise a scheme that can be used to transmit digital data at any specific angle with the help of the spectral switches controlled by the easy adjustment of one movable slit.

3.1. Spectral intensity distribution when $\theta = 0$ (on the axis)

When the observation point p is exactly at the center o of the observation plane as in figure 1, the angle $\theta = 0$ is held. Therefore the equalities $\sin(\theta) = 0$ and $\mathrm{sinc}(0) = 1$, can be substituted into equation (7) to give the spectral intensity at $\theta = 0$ as

$$I(\theta=0,\omega) = I_1(0,\omega) = A\omega^2 G(\omega). \quad (8)$$

It is found from the above equation that $G(\omega)$ is now modified by a simple function $M(\theta=0,\omega) = \omega^2$, as shown in figure 2 for two different values of Γ. The peak of the diffracted spectrum $I_1(0,\omega)$ is always blue-shifted and its magnitude is related to the bandwidth Γ. The amount of shift increases as the bandwidth Γ rises, as in figure 2. The maximum of the spectral intensity is at $\omega_{\max I} = \frac{1}{2}[1+(1+(2\gamma)^2)^{1/2}]\omega_0$, and the amount of shift is $\Delta\omega = \omega_{\max I} - \omega_0 = \frac{1}{2}[(1+(2\gamma)^2)^{1/2} - 1]\omega_0$. This behavior where the incident spectrum $G(\omega)$ is modified by the ω^2 term at $\theta = 0$ can also be found in other works with different aperture structures [3, 9, 10]. Since the peak is always blue-shifted, it is not possible to find the spectral switch effect at $\theta = 0$ and thus this direction cannot be used as a data transmission angle.

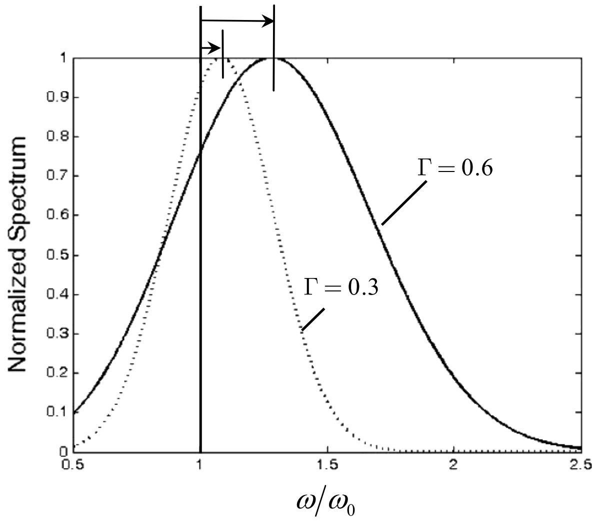

Figure 2. Spectral intensity of $I_1(0,\omega) \propto \omega^2 G(\omega)$ on the axis ($\theta = 0$) for two different bandwidths $\Gamma = 0.3\omega_0$ and $0.6\omega_0$. The spectrum is always blue-shifted. As the bandwidth γ increases, the magnitude of the peak's shift increases. (Each curve is normalized to its maximum value.)

3.2. Spectral intensity distribution when $\theta \neq 0$ (off the axis)

When the angle is off the axis ($\theta \neq 0$), equation (7) can be represented as

$$I(\theta,\omega) = B\exp\left\{-\frac{(\omega-\omega_0)^2}{\Gamma^2}\right\}\left\{\sin^2\left(\frac{b\tan(\theta)\omega}{2c}\right) \times \cos^2\left(\frac{(a+d)\tan(\theta)\omega}{2c}\right)\right\} \equiv BG(\omega)M(\theta,\omega), \quad (9)$$

where $B = 4/(\pi\tan(\theta)z)^2$ and the incident Gaussian spectrum is modulated by $M(\theta,\omega) = \sin^2(b\tan(\theta)\omega/2c)\times\cos^2[(a+d)\tan(\theta)\omega/2c]$, which is a product of the squares of the two sinusoidal functions. As mentioned in the introduction, the oscillatory behavior of the modifier functions (in this case, sinusoidal functions) is the necessary condition for the existence of the spectral switch. It is known from previous works [2, 3] that if the modulation function can redistribute the incident Gaussian spectrum into two peaks with equal heights, a spectral switch can be observed. In the following, the procedure for making the spectral switch happen at any desired angle is first explained and then how to use it to perform the data transmission by adjusting the right slit is presented in the next section. First, let us assume that the needed transmission angle is at $\theta = \theta_t \neq 0$ and we want $G(\omega)$ to be split into two symmetrical peaks with equal height at $\omega = \omega_0$. By carefully examining $M(\theta,\omega)$, we see that the above requirement can always be satisfied under the following two conditions. (1) The zero of $\sin^2(b\tan(\theta)\omega/2c)$ in $M(\theta,\omega)$ is set at $\omega = \omega_0$ for $\theta = \theta_t$; that is, $b\tan(\theta_t)\omega_0/2c = n\pi$, $n = 1, 2, 3, \ldots$. (2) The zero of $\cos^2[(a+d)\tan(\theta)\omega/2c]$ in $M(\theta,\omega)$ is also set at $\omega = \omega_0$ for $\theta = \theta_t$; that is, $(a+d)\tan(\theta_t)\omega_0/2c = (m-1/2)\pi$, $m = 1, 2, 3, \ldots$. When the above two conditions hold true, the two sinusoidal functions in $M(\theta,\omega)$ have symmetric distributions with respect to $\omega = \omega_0$ resulting in the needed symmetry property of $M(\theta, \omega)$.

Some examples are used for illustrative purposes and the following parameters are used in all the subsequent figures and numerical results: $\omega_0 = 2\pi \times 10^{15}$ rad s^{-1} and $\Gamma = 0.01\omega_0$, unless specified otherwise. If the angle is picked at $\tan(\theta_t) = 3.0 \times 10^{-3}$, then for example by choosing $n = 20$ and $m = 25$ in the above conditions 1 and 2 respectively, the parameters $b = 2.0$ mm and $a+d = 2.45$ mm can be obtained. By requiring that both of the sinusoidal functions have zero value at $\omega = \omega_0$ under the two conditions, it is found that the modulation function is always symmetric at $\omega = \omega_0$, as shown in figure 3(a) with the above parameters. Thus the modulated diffracted spectrum in the figure is split into two peaks with equal height under such a condition, with the solid dot(s) in the figure indicating the position of the spectrum peak(s). The splitting point at $\omega = \omega_0$ (marked with a small circle on the x axis) is called the phase singular point because the diffracted intensity (or amplitude) at that frequency is zero, which makes the phase singular there. If one of the conditions stated above is violated, usually the modulation function is not symmetric with respect to $\omega = \omega_0$ and the resultant diffracted spectrum will not have two equal high peaks. For example, if the value of b or $a+d$ is varied such that n or m is no longer a positive integer in the two above conditions, the spectral intensity usually cannot have a symmetric distribution, as shown in figure 3(b) for $n = 20.1$, $m = 25$ and figure 3(c) for $n = 20$, $m = 24.8$.

The angular dependence of the spectral distribution in the vicinity of $\tan(\theta_t) = 3.0 \times 10^{-3}$ can be studied as follows. Consider the case in figure 3(a), which exhibits two symmetric peaks with equal height. If the angle is varied a little to $\tan(\theta) = 3.01 \times 10^{-3} > \tan(\theta_t)$, it is found that the left peak is suppressed significantly and the spectrum peak is blue-shifted, as shown in figure 4(a). On the other hand, if the angle is decreased a little to $\tan(\theta) = 2.99 \times 10^{-3} < \tan(\theta_t)$, it is found that the right peak is suppressed significantly and the spectrum peak is red-shifted, as shown in figure 4(b). Thus when the angle is crossing the neighborhood of θ_t, the peak of the spectrum has a discontinuous jump at θ_t from blue-shift to red-shift, which is called the spectral switch, as seen from figures 3(a) and 4.

3.3. Spectral switch control via the aperture mechanism

With the analytic expression of the modulation function in equation (9), we can investigate how the spectrum changes with the movement of the right slit. Again, assume that we already satisfy the spectral switch condition at the necessary transmission angle with the same parameters as were used in figure 3(a), that is $\tan(\theta_t) = 3.0 \times 10^{-3}$, $b = 2.0$ mm and $a + d = 2.45$ mm. It is found from equation (9) that the modulation function depends on b and $a + d$ for the aperture part; therefore for convenience of discussion, it is assumed that the left slit is fixed at the position $a = 1.2$ mm and the movable right slit at $d = 1.25$ mm, as shown in figure 1(b). With equation (9), figures 5(a) and (b) show the normalized diffracted spectrum for a slight shift of the right slit to the left ($d = 1.24$ mm) and to the right ($d = 1.26$ mm) respectively.

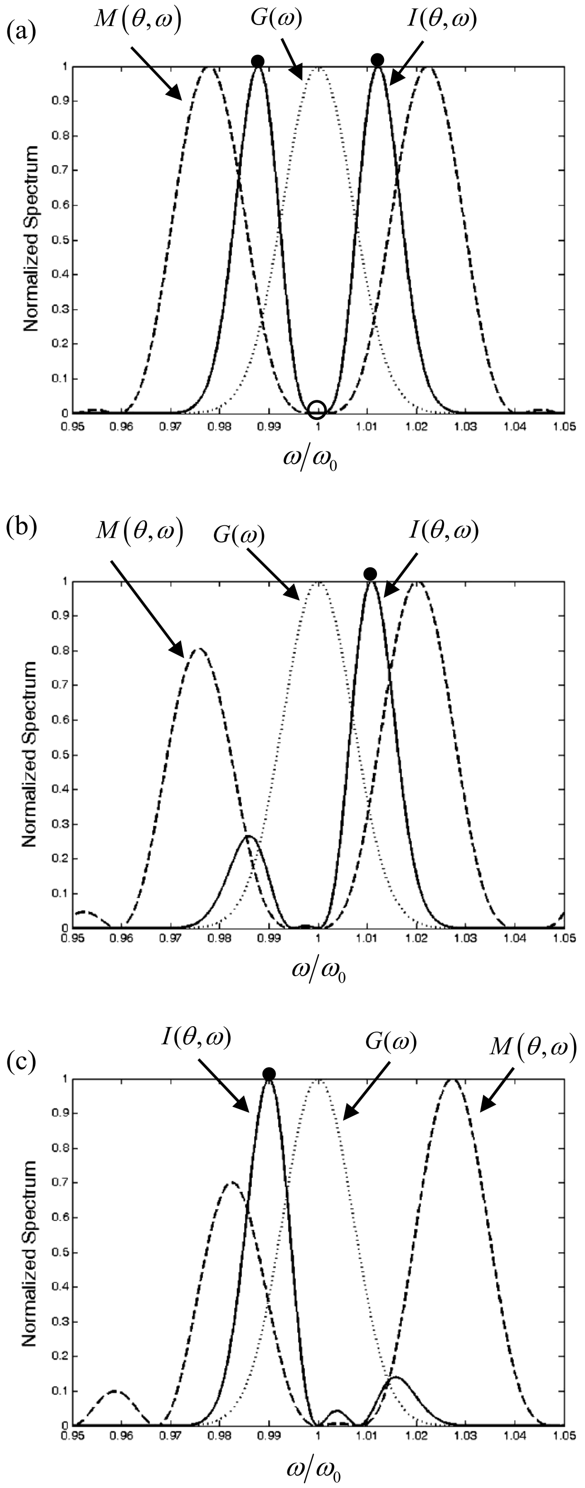

Figure 3. Normalized spectral intensity for $I(\theta, \omega)$ (solid line), $G(\omega)$ (dotted line), and $M(\theta, \omega)$ (dashed line) at $\tan(\theta_t) = 3.0 \times 10^{-3}$ with respect to different values of n or m. (a) $n = 20$ and $m = 25$. (b) $n = 20.1$, $m = 25$. (c) $n = 20$, $m = 24.8$. For all the following figures, the same curve styles are used consistently. The small solid dots in the plots indicate the position of the maximum of the spectrum.

The red-shift and blue-shift in both figures are obvious, which means that the spectral switch can be controlled simply by the slight movement of the right slit. It is interesting to note

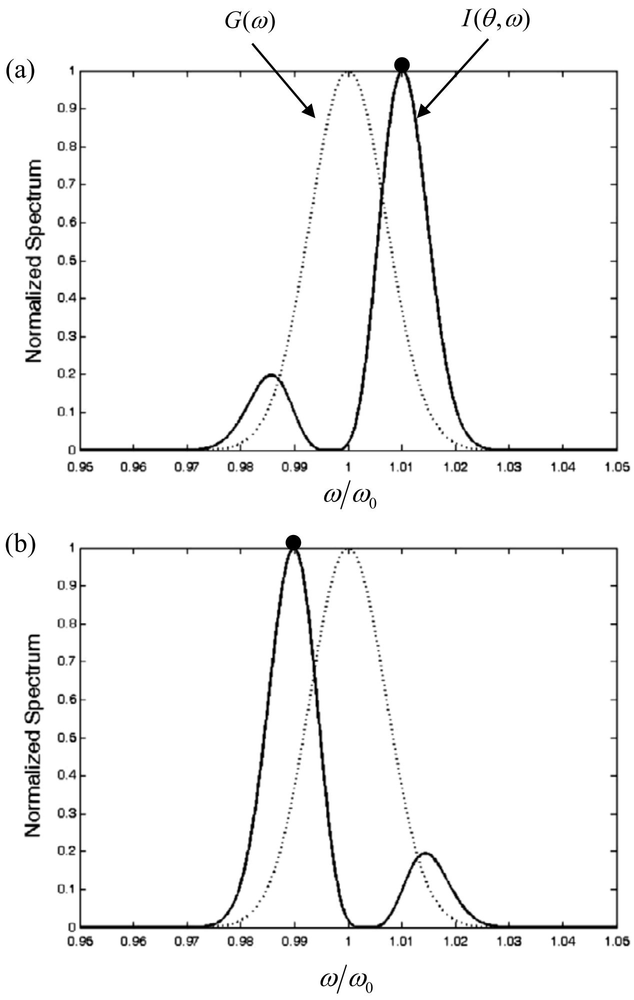

Figure 4. Normalized spectral intensity for $I(\theta, \omega)$, $G(\omega)$, and $M(\theta, \omega)$ at the vicinity of the transmission angle $\tan(\theta_t) = 3.0 \times 10^{-3}$. (a) $\tan(\theta) = 3.01 \times 10^{-3}$. (b) $\tan(\theta) = 2.99 \times 10^{-3}$.

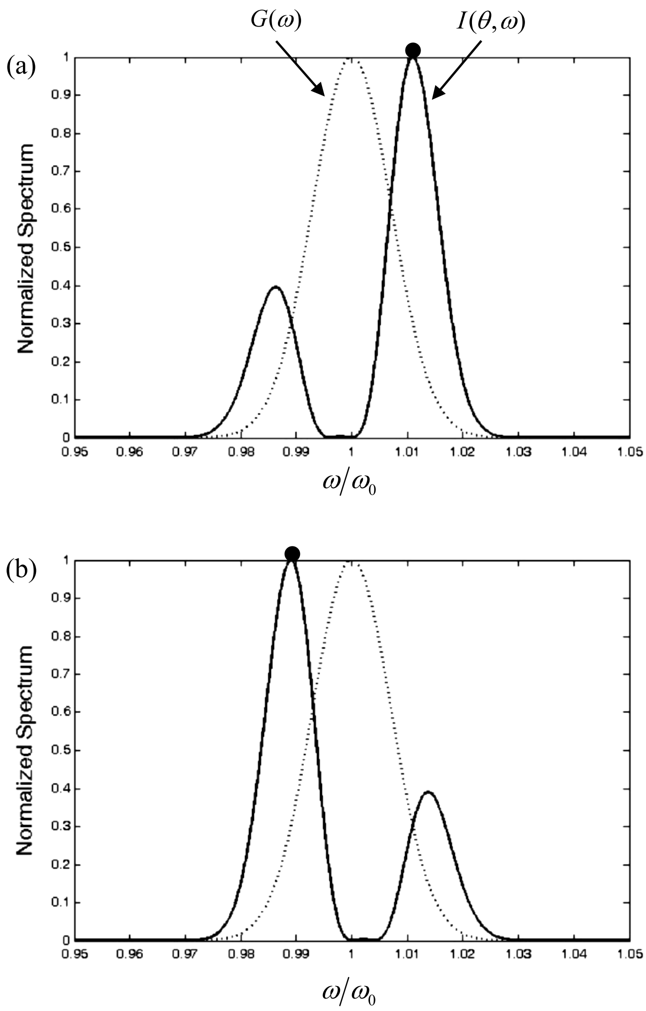

Figure 5. Normalized spectral intensity for $I(\theta, \omega)$ and $G(\omega)$ for different values of d. (a) $d = 1.24$ mm. (b) $d = 1.26$ mm.

from equation (9) that the spectral switch dependence on the movement of the slit d is periodic. For further investigation of this property, the normalized frequency shift notation is used and it is defined as

$$\Omega = (\omega_p - \omega_0)/\omega_0, \tag{10}$$

where ω_p is the frequency at which the spectrum of the diffracted spectrum peaks. This quantity is plotted as a function of the movement d in figure 6. It is obvious in this case that for every increment of 0.05 mm of the movement, there is a periodic discontinuous jump of the spectral peak from red-shift to blue-shift or conversely, where the spectral switch happens. Thus there are many choices for the distance of d, as found in figure 6, as long as the modulation function can redistribute the spectrum into two symmetric peaks with equal height at $\omega = \omega_0$. The above condition $d = 1.25$ mm used in figure 5 corresponds to the first jump in figure 6.

The spectral switches and diffracted spectrum peak's shift have been utilized in information encoding and transmission in free space [2, 3, 7–10]. In this paper, we propose another

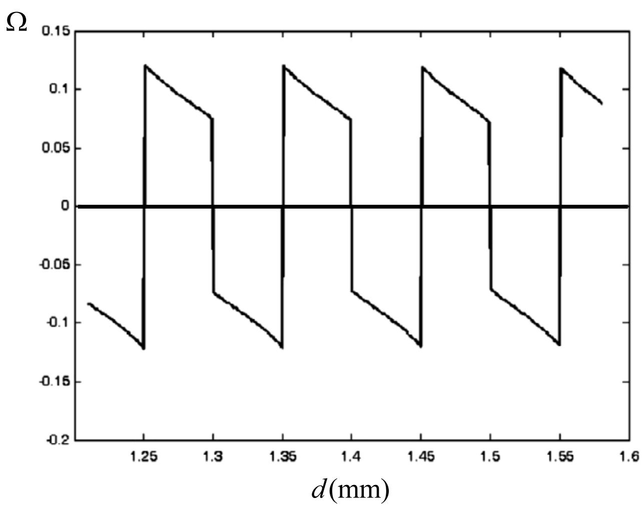

Figure 6. Plot of the normalized frequency shift Ω as a function of d. It is found that there is a periodic jump at every increment of 0.05 mm of the movement, where the spectral switch happens. The condition $d = 1.25$ mm used in figure 5 corresponds to the first jump.

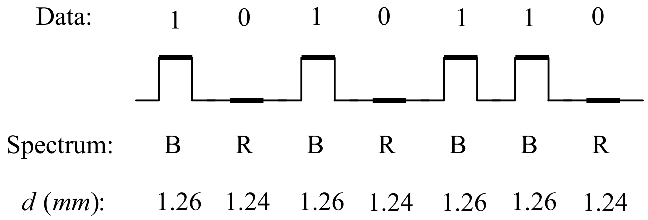

Figure 7. Illustration of the data encoding and information transmission by controlling the movement of the right slit. The blue-shift (B, for short) is associated with a bit of information, say '1', and the red-shift (R, for short) is associated with the bit '0'.

method for controlling the spectral switch by shifting one of the slits while the properties of incident light source do not need any changes. Assume that there is a set of data (as shown in the first row of figure 7) that needed to be transmitted to a position p which makes an angle θ_t with the optical axis $\overline{o'o}$. Using the strategy developed in section 3.2, the spectrum splitting into two peaks with equal height can be found and then the blue-shift or red-shift of the peak can be controlled by the movement of one slit. If we designate blue-shift and red-shift as bit '1' and '0' respectively (the symbols B and R are used to indicate the blue-shift and red-shift in the third row of figure 7), then by properly adjusting the distance d, the blue-shift or red-shift of the spectrum's peak can be controlled accordingly. Thus the data can be transmitted in free space and are detected or decoded by the receiver at θ_t. For the numerical example in figure 5, it is found that when $d = 1.24$ mm and $d = 1.26$ mm, the red-shift ($\Omega = -0.11$) and blue-shift ($\Omega = 0.11$) can be obtained respectively. In this situation, the data can be encoded and transmitted through the adjusting of the distance d, as shown in the bottom row in figure 7.

Now we can compare this scheme with another one [3] which utilized a movable central part to perform the task. An analytic presentation for diffracted spectra was derivable there; however, the transmitting angle can still only be found by a numerical method due to the complicated form of the modifier function. In this shifting slit work, we can obtain the two important conditions stated above after carefully analyzing the modifier function because of its simpler form. Since these two conditions are expressed analytically, for assuring the appearance of the spectral switch, they can be used for any selected angles and improve the feasibility of the data transmitting scheme substantially.

4. Conclusion

The following two points summarize this work. First, for any specific transmission angle, a scheme with the analytic conditions utilizing the spectral switch is devised for data transmitting, which improves the practical usage significantly. In the past, only the integral numerical results were provided and no explicit way was given for finding the appropriate choice of parameters for performing the data transmission work at arbitrary angle. Second, the aperture mechanism which uses the movement of one of the slits to control the spectral switch is shown. This aperture adjustment mechanism has the merit of easy performance compared with the light source mechanism which needs control over some properties (e.g. the spectral bandwidth or the spatial coherence) of the source.

The analytic presentation for the diffracted spectrum incident on a movable double slit is derived, which helps one to decide on the two important conditions making the spectral switch happen at the desired angle. The numerical examples illustrate the success of this scheme and its feasibility of implementation.

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